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that these conditions are also necessary. It remains, therefore, only to examine the case when  $H$  has two independent generators whose orders are not powers of the same prime number.

In this general case, we may write  $h$  in the form  $3^{a_0} p_1^{a_1} p_2^{a_2} \dots$  and observe that each of the subgroups whose orders are  $3^{a_0}, p_1^{a_1}, p_2^{a_2}, \dots$  is either cyclic or the direct product of two cyclic groups.\* The necessary and sufficient conditions that  $s_1$  and  $s_2$  can be so chosen as to generate a group which contains any one of these subgroups, as  $H$ , have been determined. If these conditions are satisfied for each of the given subgroups, we may suppose  $s_1$  and  $s_2$  so formed as to generate the group formed by establishing a  $(3^{a_0}, p_1^{a_1}, p_2^{a_2} \dots)$  isomorphism between the separate group whose  $h$ 's are powers of primes. If they are not satisfied in each instance, it follows from the equation  $s_3 s_4 s_5 = 1$  that there is no group in the infinite system under consideration which has this  $H$ . Hence this general case is included under the special cases considered above.

CORNELL UNIVERSITY, JULY, 1901.

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## ON THE INVARIANTS OF A QUADRANGLE UNDER THE LARGEST SUBGROUP, HAVING A FIXED POINT, OF THE GENERAL PROJECTIVE GROUP IN THE PLANE.\*\*

BY W. A. GRANVILLE.

IN the ANNALS OF MATHEMATICS, vol. 12, p. 82, Professor Lovett proposes the problem of finding the invariants of a quadrangle under the transformations of the six parameter group in the plane generated by the infinitesimal transformations :

$$\boxed{xp \quad yp \quad xq \quad yq \quad x^2p + xyq \quad xyp + y^2q},$$

$$p = \frac{\partial f}{\partial x}, \quad q = \frac{\partial f}{\partial y}.$$

This is the largest subgroup of the general projective group in the plane, which has a fixed point. The invariants found by Professor Lovett were

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\* Cf. *Bull. Amer. Math. Soc.*, vol. 7, 1901, p. 424.

\*\* Presented to the American Mathematical Society at its meeting, April 27, 1901.

$$c_1 = \frac{y_3}{y_4} \cdot \frac{x_4 y_2 - x_2 y_4}{x_3 y_2 - x_2 y_3}, \quad c_2 = \frac{x_2}{x_3} \cdot \frac{x_1 y_3 - x_3 y_1}{x_1 y_2 - x_2 y_1}.$$

On trial it will be found that these functions do not satisfy the differential equations of the complete system (6) on page 82 :

$$\begin{aligned} \sum_{i=1}^4 x_i \frac{\partial \phi}{\partial x_i} &= \sum_{i=1}^4 y_i \frac{\partial \phi}{\partial y_i} = \sum_{i=1}^4 x_i \frac{\partial \phi}{\partial y_i} = \sum_{i=1}^4 y_i \frac{\partial \phi}{\partial x_i} = 0, \\ \sum_{i=1}^4 \left\{ x_i^2 \frac{\partial \phi}{\partial x_i} + x_i y_i \frac{\partial \phi}{\partial y_i} \right\} &= \sum_{i=1}^4 \left\{ x_i y_i \frac{\partial \phi}{\partial x_i} + y_i^2 \frac{\partial \phi}{\partial y_i} \right\} = 0; \end{aligned} \quad (6)$$

and hence cannot be the invariant functions.

The two solutions of (6) are found to be

$$c_1 = \frac{\left(\frac{y_1}{x_1} - \frac{y_3}{x_3}\right) \left(\frac{y_2}{x_2} - \frac{y_4}{x_4}\right)}{\left(\frac{y_1}{x_1} - \frac{y_4}{x_4}\right) \left(\frac{y_2}{x_2} - \frac{y_3}{x_3}\right)} = \frac{m_{01} - m_{03}}{m_{01} - m_{04}} \cdot \frac{m_{02} - m_{04}}{m_{02} - m_{03}};$$

and

$$c_2 = \frac{\left(\frac{y_1}{x_1} - \frac{y_1 - y_3}{x_1 - x_3}\right) \left(\frac{y_1 - y_4}{x_1 - x_4} - \frac{y_1 - y_2}{x_1 - x_2}\right)}{\left(\frac{y_1}{x_1} - \frac{y_1 - y_4}{x_1 - x_4}\right) \left(\frac{y_1 - y_3}{x_1 - x_3} - \frac{y_1 - y_2}{x_1 - x_2}\right)} = \frac{m_{10} - m_{13}}{m_{10} - m_{14}} \cdot \frac{m_{14} - m_{12}}{m_{13} - m_{12}};$$

where  $m_{ik}$  is the slope of the line drawn from the point  $(x_i, y_i)$  to the point  $(x_k, y_k)$ , the origin being denoted by  $(x_0, y_0)$ . Hence the theorem :

*If a quadrangle (1234) be transformed by the Lie group*

$$\boxed{xp \quad yp \quad xq \quad yq \quad x^2p + xyq \quad xyp + y^2q},$$

*the cross ratio of the pencil of lines drawn from any vertex of the pentagon (01234) to the remaining four vertices remains constant.*

If the problem be considered from the standpoint of projective geometry, this result follows at once from the well known fact that the cross ratio of a pencil of any four concurrent lines is an invariant under any projective transformation. It is also evident that two of the cross ratios here considered are independent.